Production and Operations Management

UNIT - II
Waiting-Line Analysis

Approaches for Evaluating Capacity Alternatives

1. Cost-Volume Analysis
   - Break-even point
2. Financial Analysis
   - Cash flow
   - Present value
     - ❖ Pay-back Period method
     - ❖ NPV method
     - ❖ IRR method
3. Decision Theory
4. Waiting-Line Analysis

Waiting-Line Analysis

- Useful for designing or modifying service systems
- Waiting-lines occur across a wide variety of service systems
- Waiting-lines are caused by bottlenecks in the process
- Helps managers to plan capacity by ***balancing the cost of having customers wait in line with the cost of additional capacity***

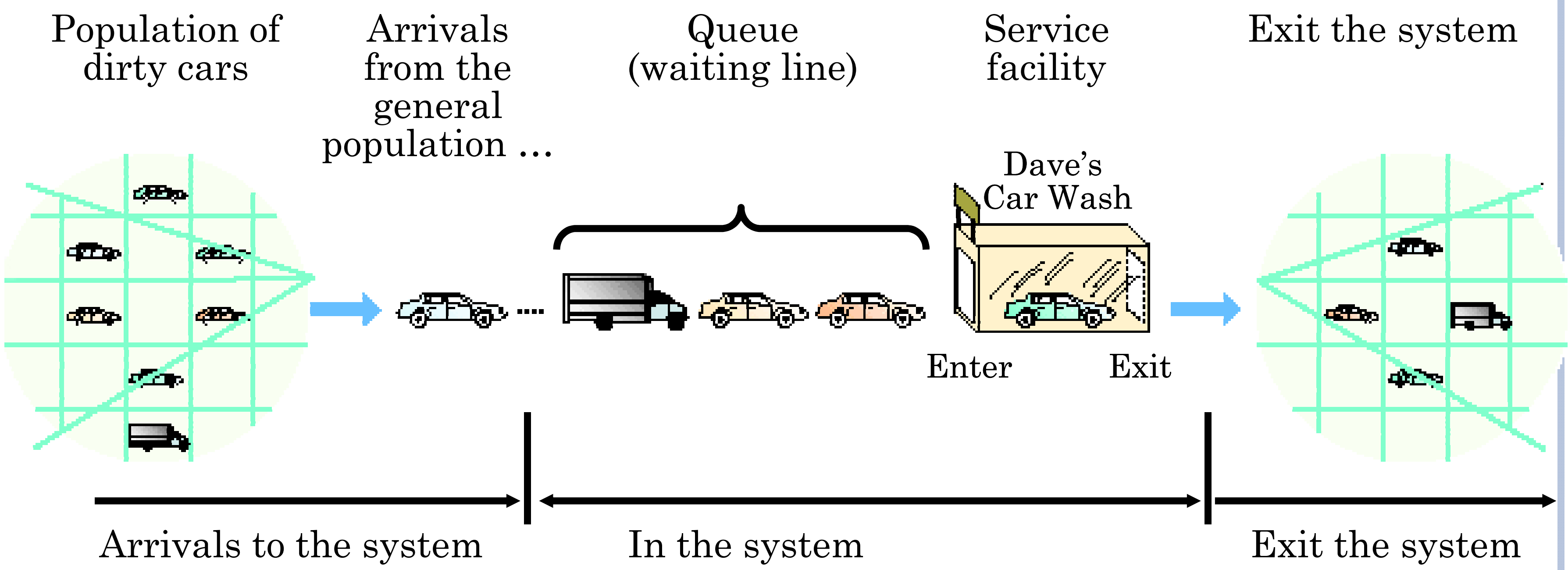
When does Waiting Line Occur?

- Demand for service exceeds the capacity to serve
- Varying arrival times of customer
- Varying service time at server

Elements of Waiting Line

- The Customer Population
- Arrival and Service Patterns
- The Service System

Parts of a Waiting Line



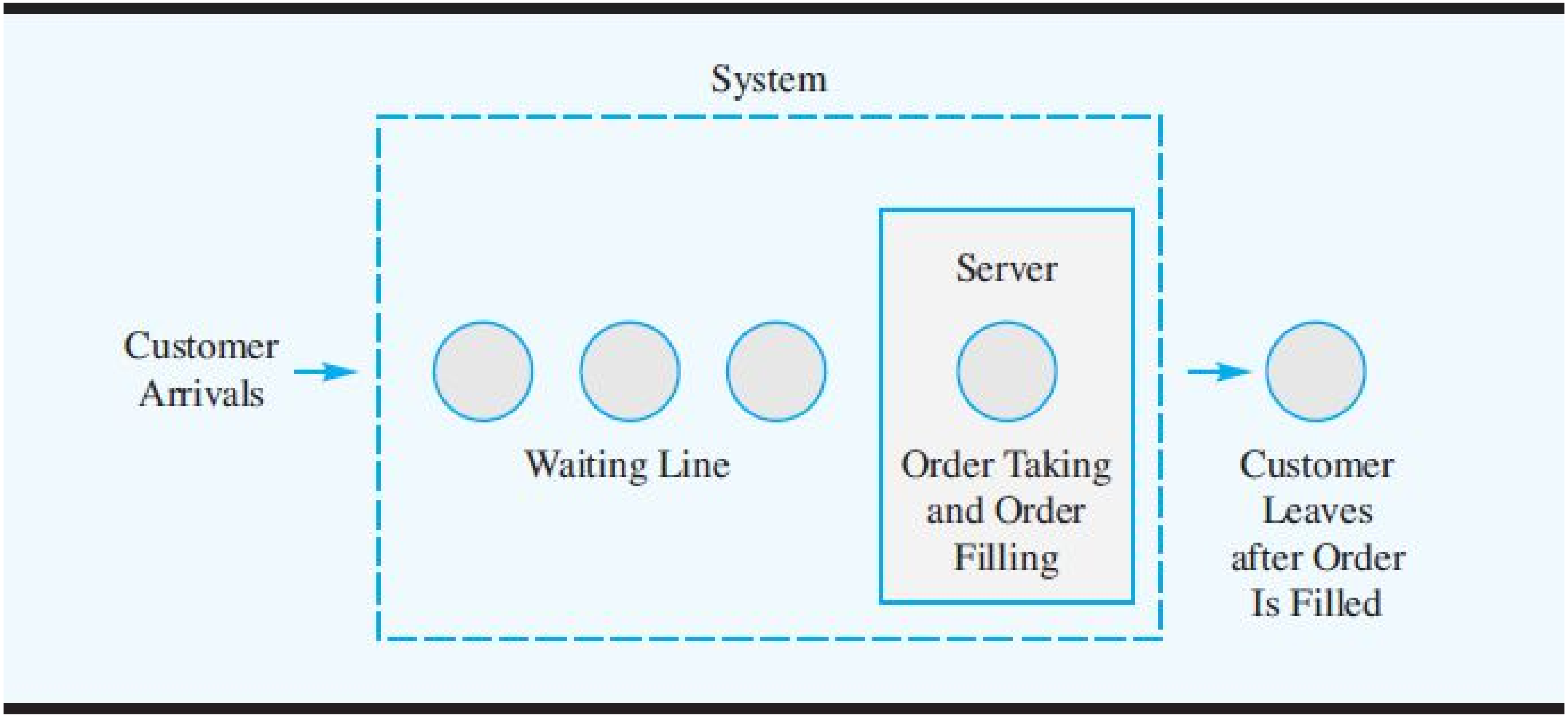
Waiting-Line Characteristics

- Limited or unlimited queue length
- Queue discipline - first-in, first-out (FIFO) is most common
- Other priority rules may be used in special circumstances

WAITING LINE PERFORMANCE MEASURES

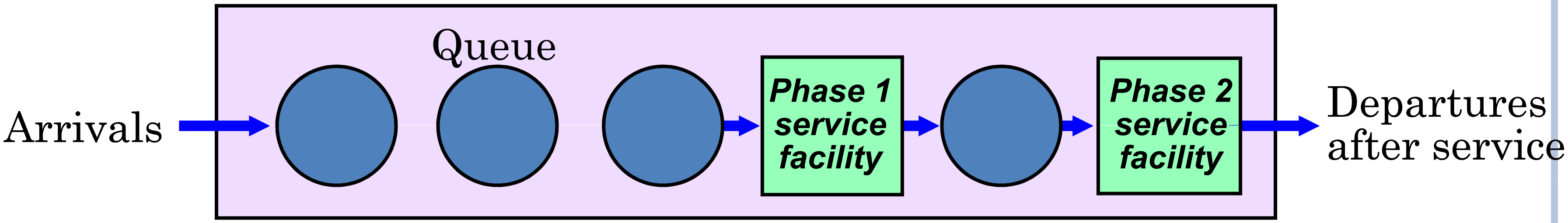
- The average number of customers waiting in line and in the system
- The average time customers spend waiting, and the average time a customer spends in the system.

SINGLE - SERVER MODEL



Single Channel, Multi phase System

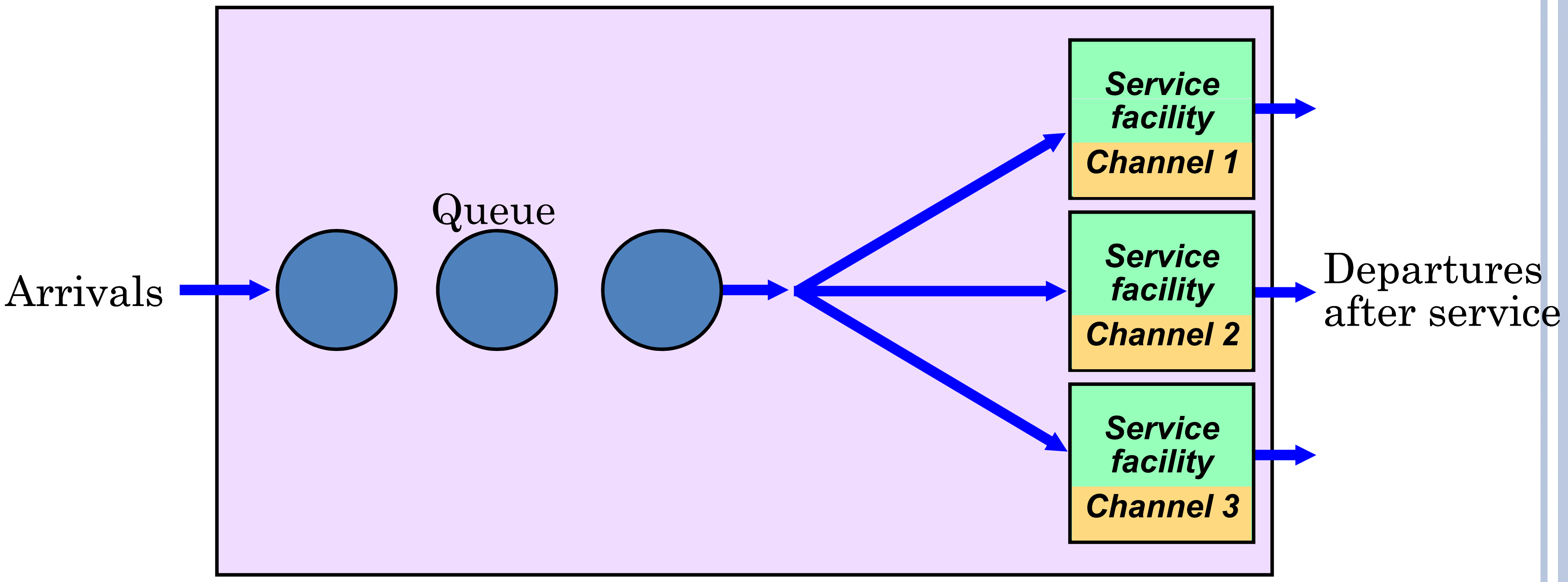
A McDonald's dual window drive-through



Single-channel, multiphase system

Multi Channel, Single phase System

Most bank and post office service windows



SINGLE - SERVER MODEL

- Arrival Rate $= \lambda$
- Service Rate $= \mu$
- Average Utilization $= p = \lambda / \mu$
- Average number of units (customers) in the system (waiting and being served)

$$L_s = \frac{\lambda}{\mu - \lambda}$$

- Average time a unit spends in the system (waiting time plus service time)

$$W_s = \frac{1}{\mu - \lambda}$$

SINGLE - SERVER MODEL

- Average number of units waiting in the queue

$$(L_q) = \frac{\lambda^2}{\mu(\mu - \lambda)}$$

- Average time a unit spends waiting in the queue

$$(W_q) = \frac{\lambda}{\mu(\mu - \lambda)}$$

Example

$\lambda = 2$ cars arriving/hour

$\mu = 3$ cars serviced/hour

$$L_s = \frac{\lambda}{\mu - \lambda} = \frac{2}{3 - 2} = 2 \text{ cars in the system on average}$$

$$W_s = \frac{1}{\mu - \lambda} = \frac{1}{3 - 2} = 1 \text{ hour average waiting time in the system}$$

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{2^2}{3(3 - 2)} = 1.33 \text{ cars waiting in line}$$

EXAMPLE CONT…

$$W_q = \frac{\lambda}{\mu(\mu - \lambda)} = \frac{2}{3(3 - 2)} = 2/3 \text{ hour}$$

= 0.66 hr = 0.667 x 60 = 40 minute average waiting time

p= λ/μ = 2/3 = 66.6% of time mechanic is busy

Example Cont…

Probability of more than k cars in the system

k	$P_{n > k} = (2/3)^{k+1}$	
0	.667 ←	Note that this is equal to 1 - P_0 = 1 - .33
1	.444	
2	.296	
3	.198 ←	Implies that there is a 19.8% chance that more than 3 cars are in the system
4	.132	
5	.088	
6	.058	
7	.039	

EXAMPLE

Customer dissatisfaction and lost goodwill= Rs.10/hour

W_q = 2/3 hour

Total arrivals= 16 per day

Mechanic's salary= Rs. 56 per day

Total hours customers spend waiting per day $= \frac{2}{3} \times (16) = 10\frac{2}{3}$ hours

Customer waiting-time cost $= 10 \times \left(10\frac{2}{3}\right) =$ Rs. 106.67

Total expected costs = Rs.106.67 + Rs. 56 = Rs.162.67